Section X2 Auto Notes **Multivariable**
("Thanks, Mr. Haugh!!!") **Calculus**

General First-Order Linear Differential Equations

Terms/Phrases/Symbols to Know: Standard Form ($\frac{dy}{dx} + P(x)y = Q(x)$), Bernoulli Equations

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Standard Form of a First-Order Linear Differential Equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where P & Q are continuous functions of x.

Solution to a First-Order Linear Differential Equation

$$ye^{\int P(x)dx} = \int Q(x)e^{\int P(x)dx}dx + C$$

with an integrating factor of $u(x) = e^{\int P(x)dx}$

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proof?

Tip: rather than memorizing the formula above, remember that multiplication by the IF $e^{\int P(x)dx}$ converts the left side of the DE into the derive of the product $ye^{\int P(x)dx}$.

Ex. Find the general solution of $xy' - 2y = x^2$

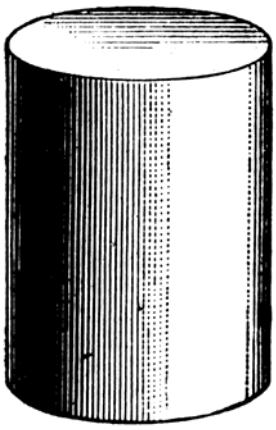
Ex. Find the general solution of $y' - y\tan t = 1,\ -\frac{\pi}{2} \le t \le \frac{\pi}{2}$

A type of Non-Linear Differential Equation: A **Bernoulli Equation** and its general solution

$$y' + P(x)y = Q(x)y^n$$

Ex: Find the general solution of $y' + xy = xe^{-x^2} \cdot y^{-3}$

Ex: A cylindrical tank contains 50 gallons of a solution that is 90% water and 10% alcohol. A second solution that is half water and half alcohol is added to the first tank at a rate of four gallons per minute. As the solution is being added, the original tank is being drained at the rate of 5 gallons per minute. Assuming the solution in the tank is stirred constantly, how much alcohol is in the tank after 10 minutes?



Ex: An object of mass m is dropped from a hovering helicopter. Find its velocity as a function of time t, assuming that the air resistance is proportional to the velocity of the object.